

**PRIFYSGOL CYMRU; UNIVERSITY OF WALES**

**DEGREE EXAMINATIONS JANUARY 2002**

**SWANSEA**

**Computer Science**

**CS 216 Theory of Programming Languages**

**EXTERNAL CANDIDATES**

**Attempt 2 questions out of 3**

**Time allowed: 2 hours**

**Students are permitted to use the dictionaries provided by the University through the invigilators**

January 2002 (External candidates)

## CS \_216. THEORY OF PROGRAMMING LANGUAGES

(Attempt 2 out of 3 questions)

### Question 1

- (a) What is a *formal language*  $L$ ? Define the *recognition problem* for the language  $L$ . Define the mathematical concept of a *grammar*  $G$  and how it defines a formal language  $L(G)$ . [5 marks]
- (b) Explain how grammars are used in the definition of programming languages. What are modular grammars and how can they assist in the definition of programming languages? [5 marks]
- (c) Define the concept of a *context-free grammar*. State the *Pumping Lemma* for context free grammars. Give an example of a property of a programming language that cannot be defined by any context-free grammar. Sketch the mathematical method used to prove such facts. [10 marks]
- (d) Give a context-free grammar that defines *one* of the following
- (i) postal addresses for a country of your choice;
  - (ii) file name formats for an operating system of your choice;
  - (iii) addresses for a type of URL of your choice.
- [5 marks]

### Question 2

- (a) Define carefully the mathematical concept of a (*many sorted*) *signature*. What aspect of a data type does a signature model? [4 marks]

- (b) Give a context-free grammar that defines the syntax of a language for defining all signatures. [5 marks]
- (c) *Sketch briefly* how to define the input-output semantics of the **while** language over all signatures. [9 marks]
- (d) What is a stream? What are *interactive systems* and what role does the data type of streams play in their modelling?
- Let  $\Sigma$  be any signature containing the booleans and natural numbers. Given a  $\Sigma$ -algebra  $A$ , construct a  $\Sigma_{\text{Stream}}$ -algebra  $A_{\text{Stream}}$  of infinite streams over  $A$ . [5 marks]
- (e) *Sketch briefly* how to define the semantics of a **while** language *with streams* over all signatures. [2 marks]

### Question 3

- (a) Let  $\Sigma$  be a many sorted signature. Define carefully the concept of a (*many sorted*)  $\Sigma$ -algebra. What aspect of a data type does an algebra model? [4 marks]
- (b) Give a signature  $\Sigma$  for the integers and an example of a finite  $\Sigma$  algebra. Does your example satisfy the same laws as the standard model of the integers? [5 marks]
- (c) Let  $A$  and  $B$  be  $\Sigma$ -algebras. Define carefully the concept of a
- $\Sigma$ -homomorphism  $\varphi: A \rightarrow B$  and
  - $\Sigma$ -isomorphism  $\varphi: A \rightarrow B$ .

Briefly, explain their role in the theory of data types.

[6 marks]

- (d) Consider the following signature.

**signature**     Machine;  
**sorts**         state, input;  
**operations**    next: state  $\rightarrow$  input  $\rightarrow$  state;  
                  output: state  $\rightarrow$  input  $\rightarrow$  output  
**endsig**

Let  $M$  and  $N$  be two  $\mathcal{M}_{\text{Machine}}$  algebras modelling two machines. Let  $\varphi: M \rightarrow N$  be a  $\mathcal{M}_{\text{Machine}}$  homomorphism. Write down the homomorphism equations for  $\varphi$ .

[4 marks]

- (e) Give Dedekind's axiomatic specification  $(\mathcal{N}, T)$  of the natural numbers. Explain how it captures precisely the abstract data type of natural numbers.

[6 marks]